# Mock Exam

In the exam you will have to answer three questions (one for each of the three topics). The questions will be roughly equally weighted $(0.3, 0.35, 0.35)$. For one topic you can choose an essay-type question. The questions could look like this:

### Question 1

Consider the random variable $Y$ with probability mass function

$$\Pr(Y = y) = (1-p)^{y-1}p,\ 0 < p < 1,\ y = 1, 2, 3, \ldots$$

You remember that $\mathrm{E}(Y) = \frac{1}{p}$ and $\mathrm{var}(Y) = \frac{1-p}{p^2}$. Let $y_1, y_2, \ldots, y_n$ be a random sample from this distribution.

a) Define the term random sample.

b) Derive the likelihood function and the log-likelihood function.

c) Derive expressions for the score, the Hessian and the information matrix.

d) Show that the expectation of the score is zero.

e) Derive the ML estimator for $p$.

f) Derive the Cramer-Rao lower bound of the ML estimator.

g) Assume $n = 10$ and $\sum_{i=1}^{10} y_i = 15$. Derive the Wald test statistic for $H_0 : p = \frac{1}{2}$.

### Question 2

a) Discuss how you check whether a GARCH model is correctly specified.

b) Discuss the leverage effect and describe a model that can account for it.

Next consider the ARCH(2) model:

$$\begin{aligned}\varepsilon_t &= \nu_t\sqrt{h_t}, \quad \nu_t \overset{iid}{\sim} \mathrm{N}(0,1)\\ h_t &= \omega + \alpha_1\varepsilon_{t-1}^2 + \alpha_2\varepsilon_{t-2}^2\end{aligned}$$

c) Write down the likelihood function for this model.

d) Derive $\mathrm{var}(\varepsilon_t)$ and $\mathrm{var}(\varepsilon_t|\varepsilon_{t-1}, \varepsilon_{t-2}, \ldots)$. Which assumption(s) do you need to make to ensure that $\mathrm{var}(\varepsilon_t)$ exists?

e) What process does the $\{\varepsilon_t^2\}$ sequence follow?

**Question 3**

Consider the ARMA(2,1) model:

$$y_t = y_{t-1} - \frac{1}{4}y_{t-2} + \varepsilon_t + \frac{1}{2}\varepsilon_{t-1}$$

where $\{\varepsilon_t\}$ is white noise with zero mean and variance $\sigma^2$.

a) Is this model stationary?

b) Is this model invertible?

c) Define the terms autocorrelation function (ACF) and partial autocorrelation function (PACF).

d) Show that $\rho_k = \rho_{k-1} - \frac{1}{4}\rho_{k-2}$ for $k > 1$.

e) Derive the two-step-ahead forecast.

f) Derive the two-step-ahead forecast error.

You analyse quarterly data on exchange rates for British pounds sterling to New Zealand dollars (xrate). You estimate three models and obtain the following R output:

```
> m1

Call:
arima(x = xrate, order = c(0, 0, 1))

Coefficients:
        ma1  intercept
      1.000     2.8329
s.e.  0.072     0.0646

sigma^2 estimated as 0.04172:  log likelihood = 4.76,  aic = -3.53
```

```
> m2

Call:
arima(x = xrate, order = c(1, 0, 0))

Coefficients:
         ar1  intercept
      0.9439     3.0111
s.e.  0.0458     0.2953

sigma^2 estimated as 0.01818:  log likelihood = 21.7,  aic = -37.4

> m3

Call:
arima(x = xrate, order = c(1, 0, 1))

Coefficients:
         ar1     ma1  intercept
      0.8925  0.5319     2.9597
s.e.  0.0759  0.2021     0.2435

sigma^2 estimated as 0.01505:  log likelihood = 25.14,  aic = -42.27
```

g) Briefly discuss the model selection criteria AIC and BIC/SBC.

h) Which model do you choose based on AIC?

i) Assume BIC/SBC selects a different model than AIC. What do you do?

### Question 4

Discuss in detail univariate GARCH models. Make sure you address the following points: estimation, model selection, model evaluation and model extensions.